BRAIN CALISTHENICS FROM THE DISTANT PAST

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"A good decision is based on knowledge and not on numbers.
Thinking – the talking of the soul with itself.
There is no harm in repeating a good thing.
Truth is the beginning of every good to the gods, and of every good to man.
Knowledge without justice ought to be called cunning rather than wisdom."

--Plato, Greek Polymath, circa 428 – 347, BCE

In our preceding two episodes, we explored some lessons from the distant past. Writing this column, along with working on case materials for clients, generally keeps my (Dr. Sase's) brain in gear. However, too much of good things can lead to getting stuck in mental muck. Therefore, for this episode, I have chosen to share some of the ongoing brain exercises that allow me to transcend the New Normal. In this, what we can call "Fun with a Purpose," I wish to focus on some activities that remain interesting and entertaining while maintaining brain strength and general skills. These are the skills that helped you, the reader, to get into and then to graduate from Law School, to pass the Bar Exam, and finally to establish your career in the profession of Law.

What Does it Take?

According to the American Bar Association (ABA), successful Law students and lawyers take courses that develop the following skills. The top seven skill groups identified include 1) Analytical methods and problem-solving, 2) Critical reading, 3) Professional writing, 4) Oral communication and listening, 5) General research skills, 6) Task organization and management, and 7) Public service and promotion of justice.

In her article "Choosing Your Major for Prelaw," Pre-Law advisor Carol Leach compiled a ranking of the top twenty undergraduate majors that helped many of you to achieve the highest Law School acceptance rate several years ago. The top seven majors include 1) Physics, 2) Philosophy, 3) Biology (specialized), 4) Chemistry, 5) Government, 6) Anthropology, and 7) Economics (huzzah!) (http://lawschoolnumbers.com/application-prep/Choosing-Your-Major-for-Prelaw).

A decade ago, College Consensus reported the seven foremost undergraduate majors and their points above average on the LSAT in their article "Majors with Highest Above Average LSAT Score." These include 1) Physics/Math (+9.0), 2) Economics (+6.4), 3) Philosophy/Theology (+6.4), 4) International Relations (+5.5), 5) Engineering (+5.2), 6) Chemistry (+5.1), and 7) Government (+5.1) (https://www.collegeconsensus.com/features/best-degrees-for-law-school, 2008).

Let us condense these twenty-one observations into what we can call “The Strengths of Analysis and Problem Solving of Philosophical and Theological Issues Using the Powers of Physics, Mathematics, and Economics.” Perhaps you may want to share and to discuss these observations with younger members of your own family who may be considering a profession in Law.

As we grow older, our minds get cluttered with the details of our professions and everyday life. Therefore, our minds may continue to thrive through a regular mental exercise of a higher nature and order. My favorite activities include finding, exploring, and understanding ideas from the ancient past from around the world. In Western Thought, what better place to begin than with the Greek philosopher/mathematician Pythagoras (c. 570 – c. 495 BCE) and Plato, who provided our opening quote? In our preceding pair of episodes, we considered their work, which reaches back in history and advances through thoughts of the past two-and-a-half millennia. Therefore, let us leap headfirst into Plato and swim to the legendary island of Atlantis.

The Urban Allegories of Plato

In *The Republic*, along with *Timaeus and Critias,* Plato presents us with four urban allegories to help us to ponder the cosmology and nature of the universe and daily living. His first two tales focus on visions of the City of Athens, which he envisions as a hill that divides into simple sections. Plato cuts the mountain in the middle vertically to form two levels. Simultaneously, he splits his mountain site of Athens into two sloping sides.

In this second of Plato’s urban allegories, the layout of Magnesia, an actual ancient city, bore little resemblance to that in his story. Plato describes his version as two concentric circles with twelve equally spaced highways that extend radially from a central capital city with each road passing through a market/temple area at the halfway point and continuing to military garrisons at the outer border.

Plato’s allegory of the City of Atlantis remains the most challenging of his four tales. He presents Atlantis as a “city of excess” as he provides a copious amount of numeric symbolism in his story. For years, I took the description at face value. However, I gradually began to look deeper, reading the works of authors such as Ernest G. McClain, Francis M. Cornford, and Robert S. Brumbaugh. I did not include Atlantis in my past writings because of the complexity and general lack of relevant economics at the time. However, as I became more enlightened about the math of Pythagoras and his predecessors, I began to find deeper meaning in the Atlantis allegory.

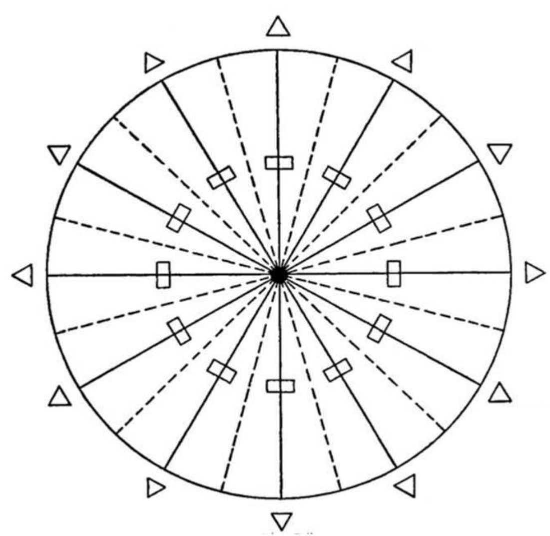
Pythagorean Mathematics

Exploring Pythagorean mathematics turned into a routine of excellent mental exercise that has helped me to clear my brain for other mathematically based work, such as my determinations of economic losses for legal matters. I have found that returning to the lessons of Pythagoras and the mathematical allegories of Plato has helped me to maintain a higher level of vision while strengthening my mental discipline for working with complex numeric databases.

Therefore, I have decided to share these mental exercises with the legal community and others. I have included a series of illustrations in my walk-through of these exercises. I recognize that the newsprints of the visuals may pose difficulties for some readers, so I have posted higher resolution versions online that those interested may download freely from www.saseassociates.com/pdf.html.

So, without further ado, let us walk through the brain exercises that have served me well over a few decades. Let us begin with the relevant basics of Pythagorean math that will lead us to our results. It remains debatable as to whether or not Pythagoras developed these ideas himself or learned them from others. Conservatively, the current discussion among academic philosophers suggests that he absorbed much of his basics as an initiate and student at the Temple School at Giza. Pythagoras learned much of his advanced math from the ancient priesthood of Giza as well as from fellow students who had journeyed to Egypt from India, Babylon, and other centers of ancient knowledge. See Leon Crickmore, "A Possible Mesopotamian Origin for Plato's World Soul," *Semantic Scholar*, 2016 and Eugene Afonasin and Anna Afonasina, "Pythagoras Traveling East: An Image of a Sage in Late Antiquity." *Archai* 27, 2019.

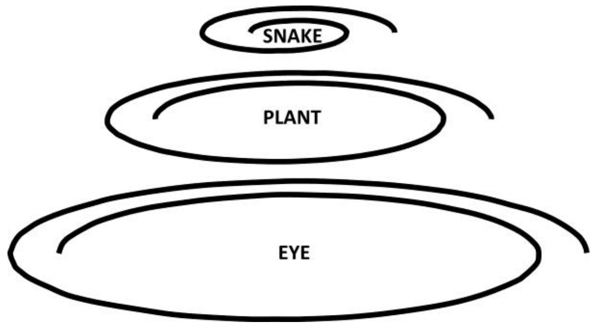
In our previous episode, we explored elements of Pythagorean-Plato analysis, which included the less-complicated model of the City of Magnesia that served in our thematic discussion of sufficient affluence within a sustainable economy.



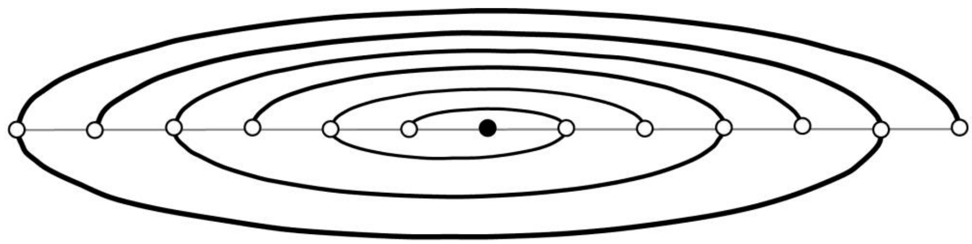
Magnesia

Exercise One

We need to consider the Pythagorean method of creating a progression of ratios that are apparent in the conceptualization of the City of Atlantis. This element builds upon the mathematical products using the exponential powers of 2 and 3. The most straightforward explanation of this construction relies upon drawings of three successive spirals that allow us to set up a simple table of products and ratios.

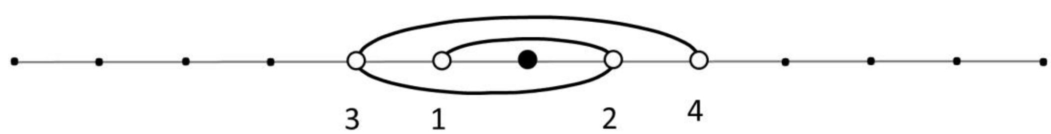


For simplicity of reference from the smallest to the largest, let us refer to these three spirals as the Curled Snake, the Blooming Plant, and the Human Eye with an eyelid. For assembly, we start with a line of thirteen equidistant points. Leaving the centermost point untouched, we use the remaining dozen points for our construction. When grouped, they appear as three concentric spirals.



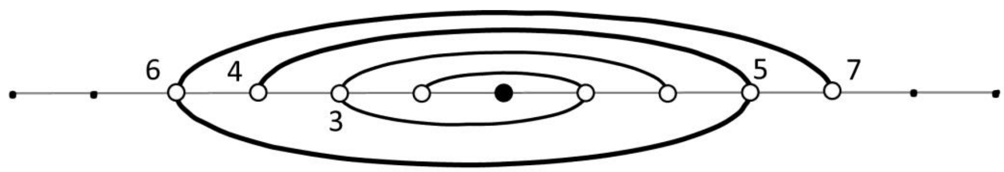
SNAKE, PLANT, & EYE

We use this composition to generate the specific numeric sequence that provides a progression of exponential powers for the numbers 2 and 3. We begin our construction at the point just to the left of the center to create the inner spiral of the Snake. We draw our Snake from point one across-right to point two, back across-left to three, and, finally, across-right to position four.



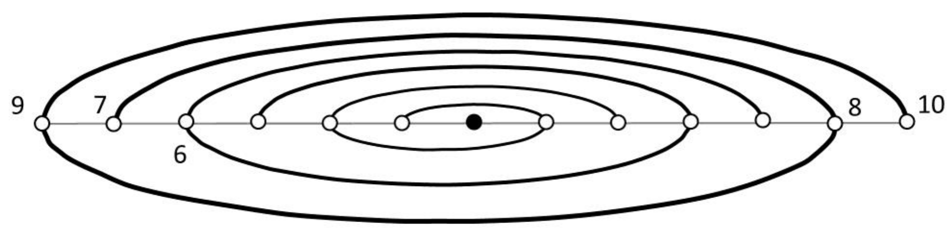
FORMING THE SNAKE

We construct the Plant by adding a second point four left of position three of the Snake. Crossing to point five at the right, we circle back to six on the left. Finally, we travel to point seven on the right.



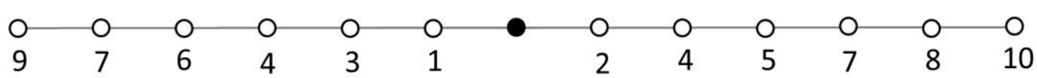
FORMING THE PLANT

We complete our construction by drawing the Human Eye. From a second number seven left of point six, we crossover right to eight and circle back to nine. We complete the Eye by traveling to point ten on the right.



FORMING THE EYE

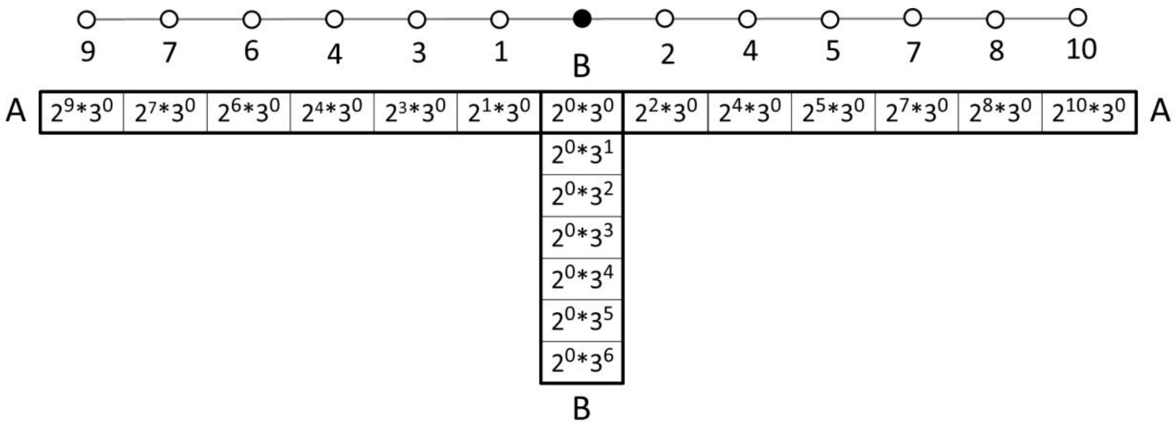
Hurray! We have successfully finished the first exercise of our Pythagorean-Plato series. Now we possess the primary sequence of the exponential powers of 2 that we need to develop our Pythagorean progression of 13 points.



EXPONENTIAL POWERS OF 2

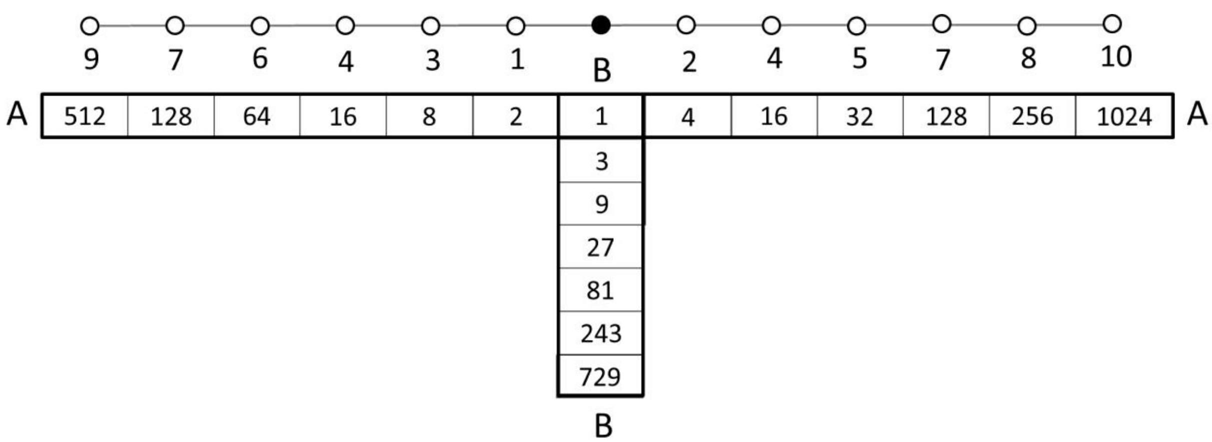
Exercise Two

Let us move along to our second exercise in which we assemble a T-Chart containing the powers of the numbers 2 and 3. Our first exercise gave us the exponents of the number 2. From left to right in row A that forms the bar of our T-Chart, we have 2^9, 2^7, 2^6, 2^4, 2^3, 2^1, 2^0, 2^2, 2^4, 2^5, 2^7, 2^8, and 2^{10}. After we calculate these thirteen values for the number 2, we will multiply each one by 3^0, which of course, equals the value one as it would for any number taken to the zero power. In the column at the center, the number 2 remains at the exponent zero. However, the corresponding exponents of 3 increase as we descend column B. As a result, we get 3^0, 3^1, 3^2, 3^3, 3^4, 3^5, and 3^6.



EXPONENTIAL T-CHART

Completing this second exercise, we calculate each of the values of 2^i*3^j to arrive at the amounts needed for our third exercise.



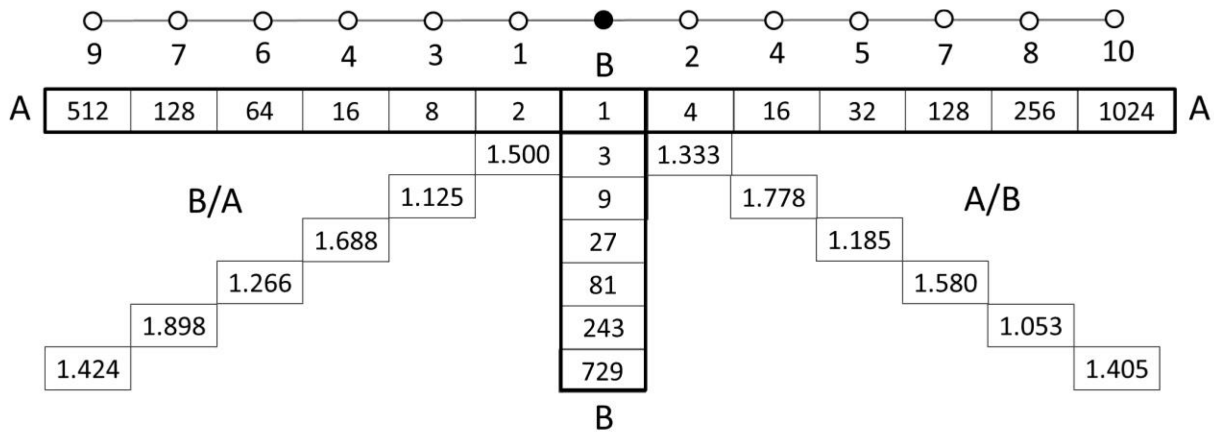
T-CHART OF PRODUCTS

Exercise Three

Our third exercise involves simple division that most of our readers learned in elementary school. I am guessing that many of us have either children or grandchildren in school (which may be distance learning at home for a while longer). These exercises may offer some intergenerational fun within your family circle.

The quotients of this exercise will provide the basic building blocks for the construction of the Atlantis Allegory developed by Plato. The critical feature produced that differentiates this model from the other three urban allegories of Plato is known as the Pythagorean Comma, which is the gap that exists between the two middlemost values.

Let us form two diagonal sets of boxes that extend from the box containing the number 1 at the top of column B. We have divided each of the values in column B by the respective values along the left side of row A on the left-hand side of the table. For example, the number 3 divided by 2 equals 1.5, and 729 divided by 512 equals 1.424. Similarly, on the left-hand side of the table, we divide each of the values along the right side of row A by the respective values in column B. For example, the number 4 divided by 3 equals 1.333 and 1024 divided by 729 equals 1.405.



T-CHART OF RATIOS

Now we have a payoff from our first three exercises. By copying the values from the two diagonals of our T-Chart to a standard 360-degree compass, we get a set of equitably spaced radii. However, the closely paired radii pointing toward the six o'clock position (180 degrees) creates a split pair. The narrow gap of this pair provides the entryway to the Atlantis of Plato. This gap, the Pythagorean Comma, forms the navigable channel from the outer boundary of Atlantis to its central island.

PLATO'S ATLANTIS

Exercise Four

Let us move onward to our fourth exercise. Whether or not a real City of Atlantis ever existed, the allegorical description by Plato has influenced most of the drawings made of Atlantis throughout modern history. However, Ernest G. McClain, a music professor at the City University of New York to whom we referred earlier, discovered that the Atlantis of Plato contains a remnant of an older form of tuning musical instruments by intervals rather than by our modern standard of equal temperament. Having tuned pianos and guitars while playing classical, rock, jazz, folk, and blues music throughout my lifetime, I understood what McClain described. Traveling full circle on this matter, I decided to confer with Professor McClain by phone before he passed. I came to rely upon his analysis, which used microtonality derived from the inferred mathematics as presented by Plato across what appears to be thirteen octaves (in which the numerical values double at each octave).

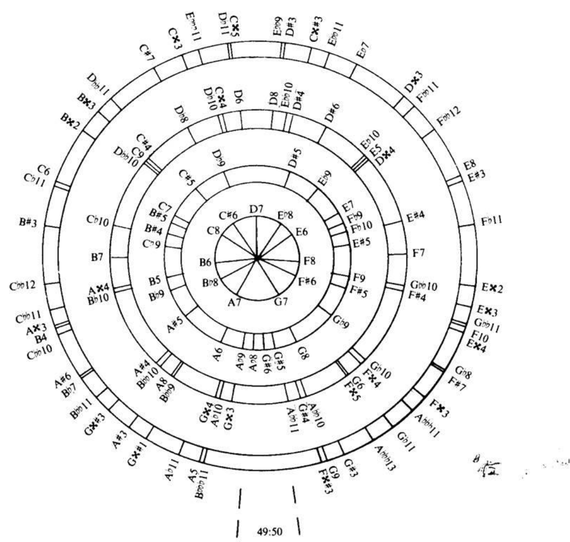


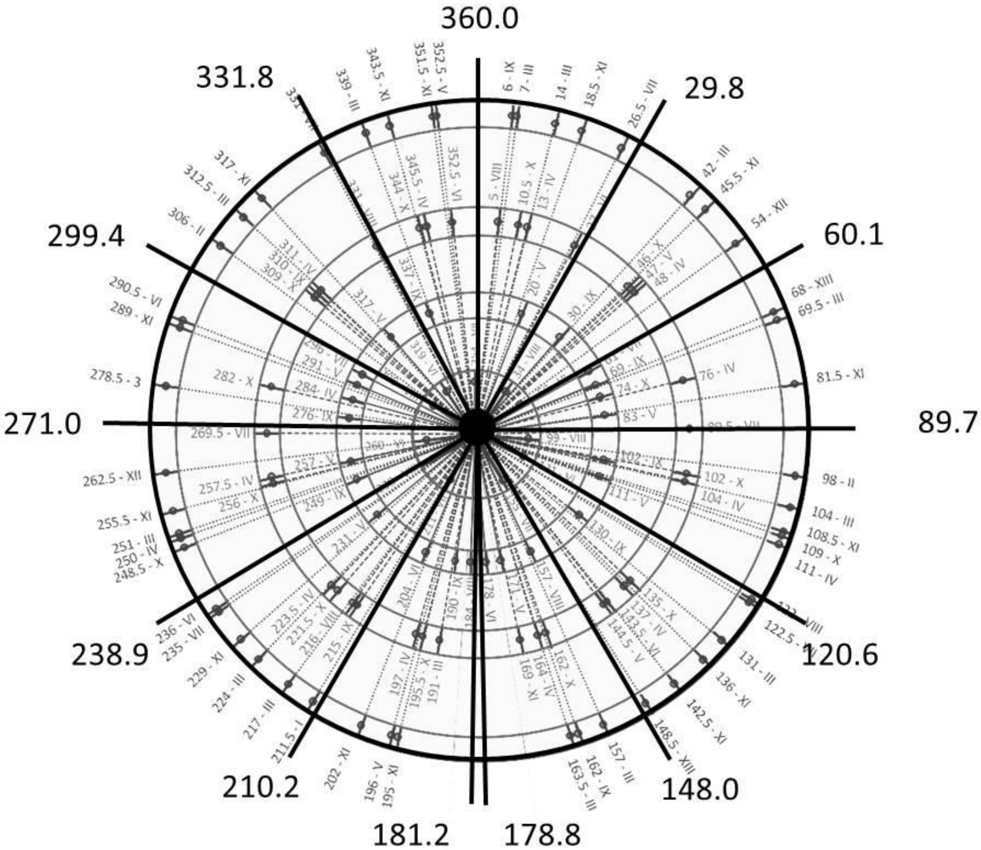
FIGURE 30
The Islands of Atlantis

McCLAIN'S ANALYSIS

Math Is Music / Music Is Math

In Plato's tale of Atlantis, 121 different tones appear across thirteen octaves, with no two octaves being the same. However, some very unusual numerical patterns appear throughout his tale. Given that Plato considered math as music and music as math, this route presents a worthwhile journey.

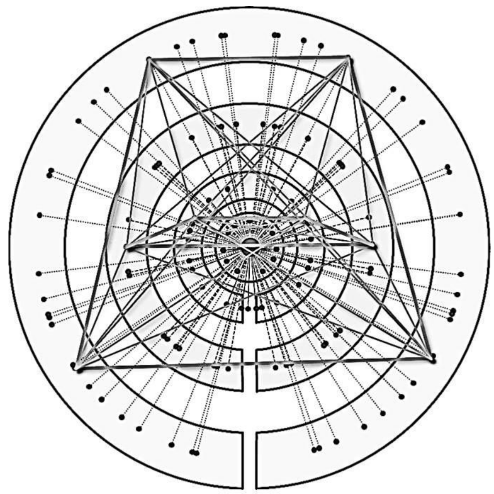
I started by converting the microtones established by McClain to degrees around a compass delineated into three rings and the central island of Atlantis. Also, I noted the incremental octaves expressed by McClain.



PYTHAGOREAN-PLATO

The approach of using a 360-degree circle presented limitations, since many of the microtones fell in between degrees. Historical records suggest that Plato obtained his advanced education at Alexandria, Egypt, which remained a hub for scholars throughout the Mediterranean area, the Near East, and the Far East. Having an acquaintance with scale systems of 17, 22, 24, and 72 intervals, I consider the probability that Plato and Pythagoras may have developed a knowledge of sound that extended beyond the Grecian world. In addition to these theories, a matter of convenience suggests doubling the number of intervals to 720 to gain a more satisfactory degree of accuracy for the mathematics needed.

Following a lengthy cursory review of the 121 notes/frequencies/vibrations in the Atlantis allegory, I found that the middlemost seventh (VII) level is unique and very symmetrical.



VII

However, in exploring the other twelve levels, I encountered several anomalies. Expanding my inquiry above and below the seventh level, I discovered a wide variation in the number of points that Plato placed on each of the other levels. I found that a definite symmetry appears between pairs of the remaining twelve levels. For example, level XIII at the top matches level I at the bottom. This pattern remains consistent through classes VIII and VI that exist immediately above and below level seven (VII).

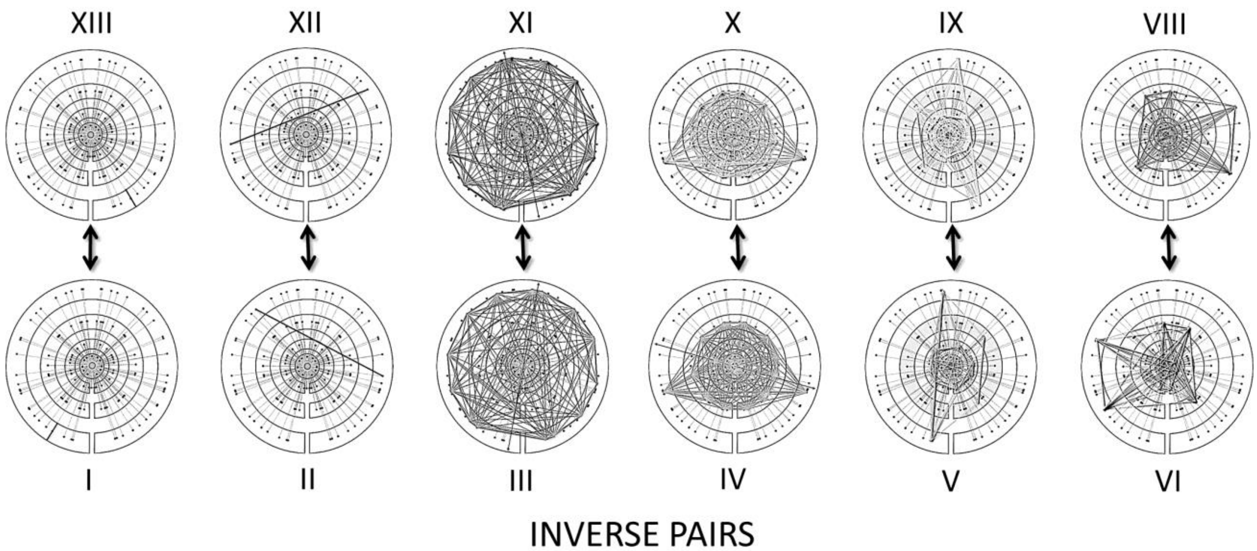
	XIII	XII	XI	X	IX	VIII	VII
UPPER	1	2	15	14	12	11	11
LOWER	1	2	15	14	12	11	
	I	II	III	IV	V	VI	

DISPERSION OF 121 POINTS

As Above, So Below

This phase of analysis provides a great mental exercise in and of itself. For those looking for more fun, connecting the dots reveals even more "secrets" that Plato produced.

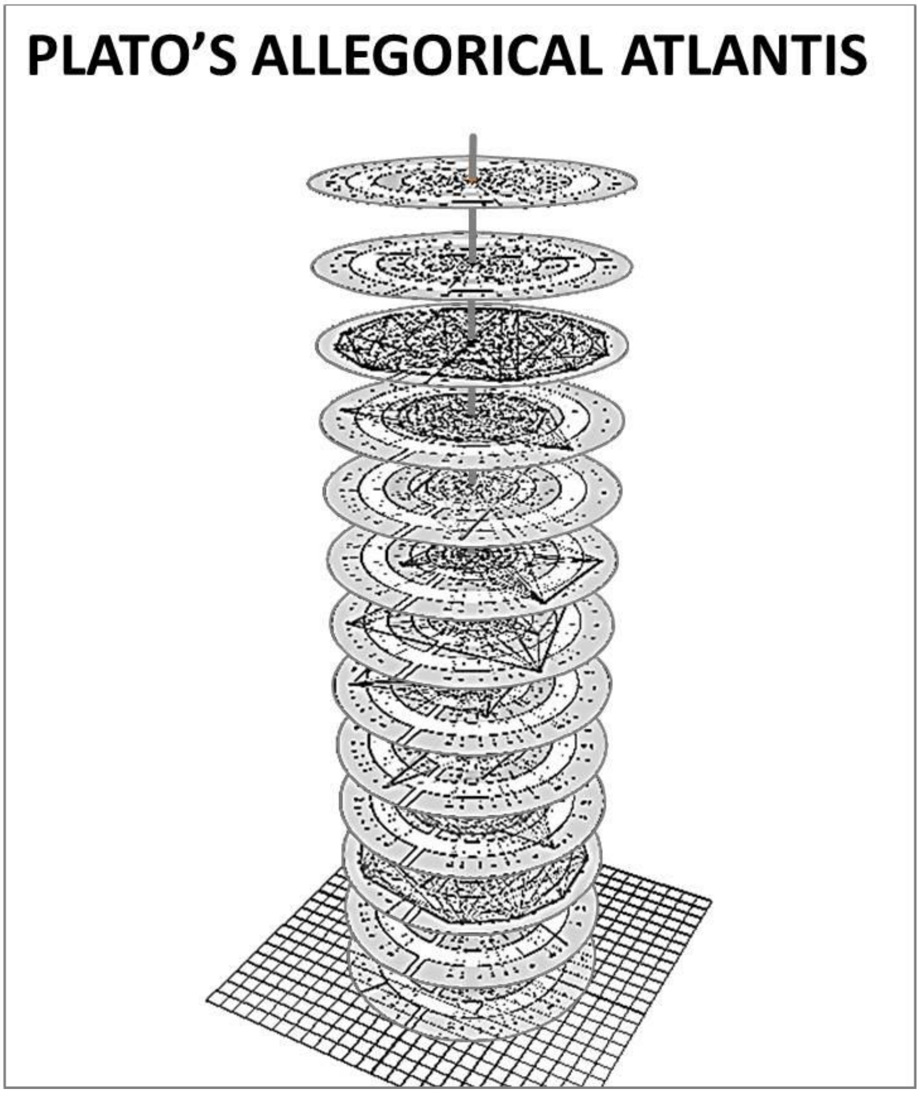
The patterns on his Atlantian levels form six pairs of mirror images. Inversely flipping levels VIII through XIII to the left creates patterns that mirror those on levels I through VI. The designs that the math of Plato produce on levels above the unique seventh level reflect in the corresponding levels below. As above, so below. At this point, I began to wonder whether Plato intended to produce a "simple" allegory about a city of excess or modeled something entirely different and more esoteric.



Exercise Five—The 500 Pound Bench Press

Given the details outlined above by Plato, we move to a three-dimensional interpretation of the Platonic model. As rendered by Ernest McClain, the primary platform includes three concentric circles of land with water in between, a navigable channel that cuts through these three rings, and a central island on which Plato locates a temple. In this depiction of Atlantis, I replaced microtonal musical pitches rendered by McClain with an exacting pattern based on 720 radii (doubling 360 degrees on a compass). As mentioned earlier, 121 position points conform to the description by Plato.

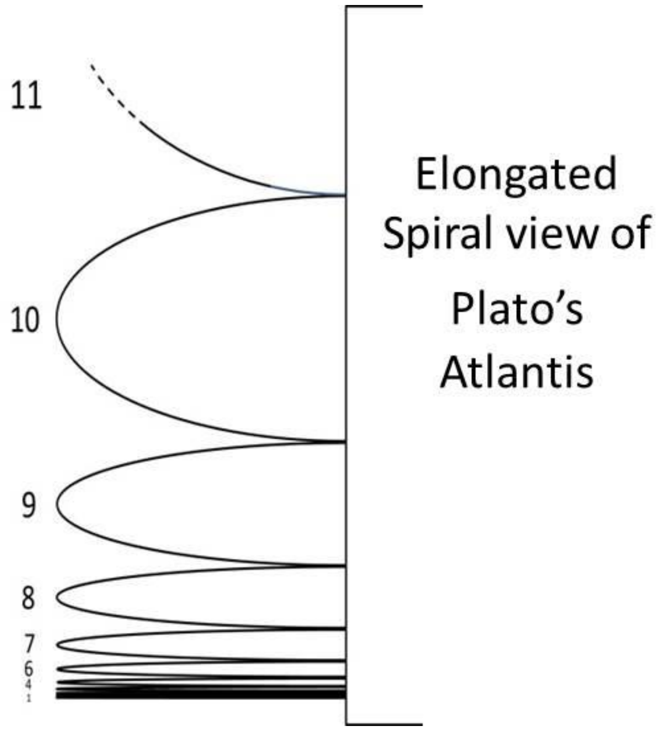
On this sketch (for which you may download a higher-resolution version in color), each of the four landmasses contains a different number of points clustered in groups on the left-hand side. They form near-mirror images of the patterns on the right. The middle island holds 11, while the smallest ring contains 24, the middle ring 36, and the outer ring 50. However, Plato broke and distributed these point-groups across the 13 levels. The most significant number of points (15) appears on the third level from the bottom and the third one from the top.



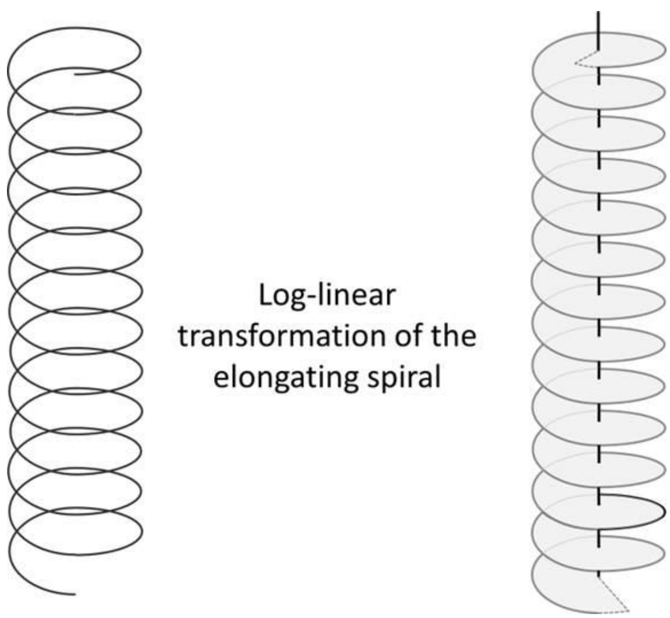
This simplified view of the thirteen levels of Atlantis reminds me less of a city and more of a fantastic direct-energy technology created by Nikola Tesla.

Some Afterthoughts

The preceding model of thirteen flat tiers allows us to create 3-D practical models using old CDs, DVDs, or plastic storage-container lids. However, if we follow the progression developed by McClain using a continuous progression of sound (music) frequencies, we might consider modeling Atlantis as an elongated spiral. For purely mathematical work, this approach allows us to explore the sound and light ranges of frequencies with greater precision using Hertz and Terahertz that are separated by a range of forty doublings (octaves).



Nevertheless, we can choose to use a log-linear transformation of the elongated spiral. This transformation allows us to simplify our mathematical calculations in a spreadsheet and on a 3-D graph. We can perform various calculations and do other activities, such as studying the 7,200 intervals that exist between the values of 1 and 2 across the thirteen levels of the Allegorical City of Atlantis developed by Plato. Also, this shape resembling the Archimedes Screw facilitates calculations by using a profile that closely parallels the stack of thirteen plains with which we began our discussion.



Takeaway

We hope that our readers who quarantine, cocoon, or work from home as we survive the Pandemic of 2020 remember not to watch too much cable news or binge on reruns of *Friends, The Sopranos, Game of Thrones* (or whatever else floats your boat). As the New Normal becomes Normal, we hope that all of us remember to keep our minds and well as our bodies in shape through activities that also feed our souls. To that end, we hope that the brain calisthenics of Pythagoras and Plato described above will help our readers to do just that. As a soundtrack to the exercises, we suggest such songs as "Those Were the Days" by Cream (*Wheels of Fire,* Atlantic, 1968) and *Atlantis* by Donovan (*Barabajagal,* Epic, 1969). Enjoy!